Mixed-mode load transfer in the fibre bundle model of nanopillar arrays

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Model

Within the framework of the fibre bundle model [1, 2] we explore the effect of mixed-mode load transfer [3] in two-dimensional arrays of nanopillars.

We consider the system as a set of N longitudinal nanopillars located in the nodes of the supporting lattice. Each pillar is characterised by its own strength threshold to an applied axial load. Pillar-strength-thresholds σ_{th}^{i}, $i = 1, 2, ..., N$ are quenched random variables distributed according to Weibull distribution

$$P(\sigma_{th}) = 1 - \exp[-(\sigma_{th}/\lambda)^{\rho}] \qquad (1)$$

In this work we assume $\rho = 2$ and $\lambda = 1$.

Loading process is realised by two different (but also equivalent) procedures: sudden loading (application of finite force) and quasi-static (gradual) loading. In the case of quasi-static loading the external load F is gradually increased up to the complete failure of the system. Initially system is unloaded and intact. Then the load is uniformly increased on all the working pillars just to destroy the weakest one. Then, the increase of external load is stopped and load from destroyed pillar is transferred to intact pillars. The load redistribution may lead to subsequent pillar failures which can provoke next failures. A stable state is achieved if the load redistribution does not cause any failures. In such a situation, the external load has to be increased again. The above described dynamics is continued until destruction of all pillars.

Sudden loading of the system is realised by application of an external force F which is kept constant during the entire loading process. Due to uniform loading, in the moment of application of load F the load per pillar is $\sigma = F/N$, so all the pillars with strength thresholds smaller than σ are immediately destroyed. Then, the load transfers may lead to next failures. This procedure lead to a stable state of the system which is either partially or fully destroyed. The third possibility is that the system is intact - only if σ is not greater than σ_{th} of the weakest pillar.

Load transfer rule

As a load transfer rule we apply mixed-mode load sharing with weight parameter g. In this scheme, when a pillar fails, fraction g of its load is transferred locally and the rest $(1 - g$ fraction) is distributed globally. Therefore, the mixed-mode load sharing is an interpolation mechanism between the GLS and LLS - $g = 0$ corresponds to the GLS rule and $g = 1$ represents pure LLS rule.

Quasi-static loading

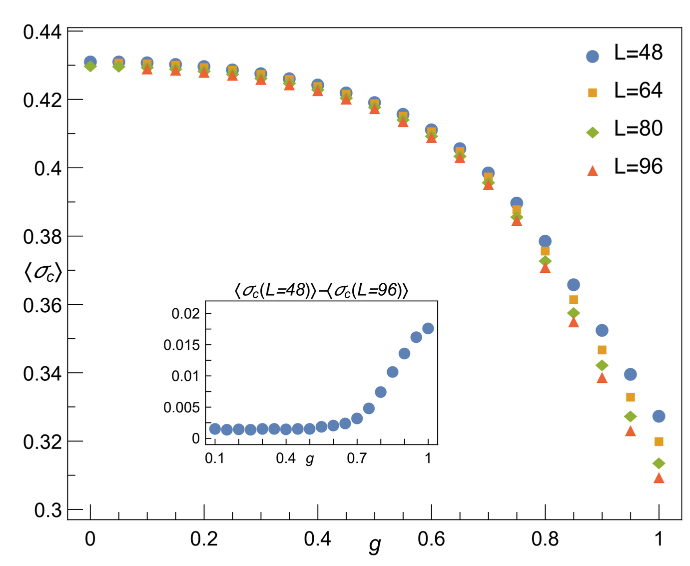

Figure 1:The mean critical load σ_c as a function of weight parameter g for different system sizes. In the inset we show the results of subtracting $< \sigma_c >$ for $L = 48$ from $< \sigma_c >$ for $L = 96$. L is a linear size of the system.

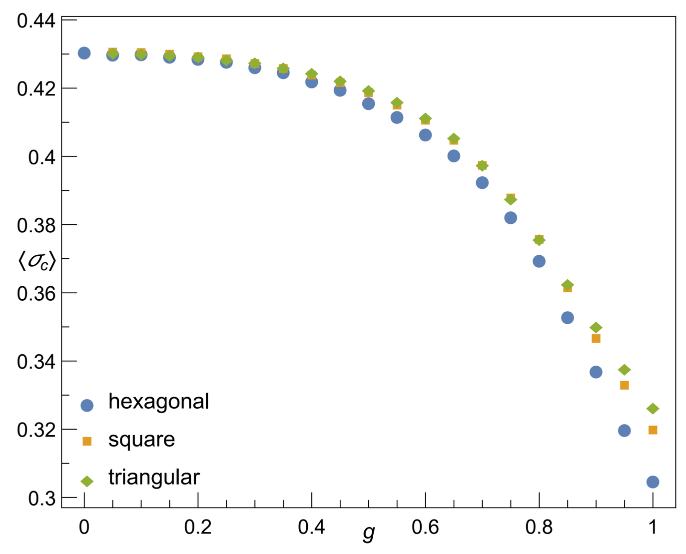

Figure 2:The mean critical load σ_c versus weight parameter g for different system geometries and $N = 64^2$.

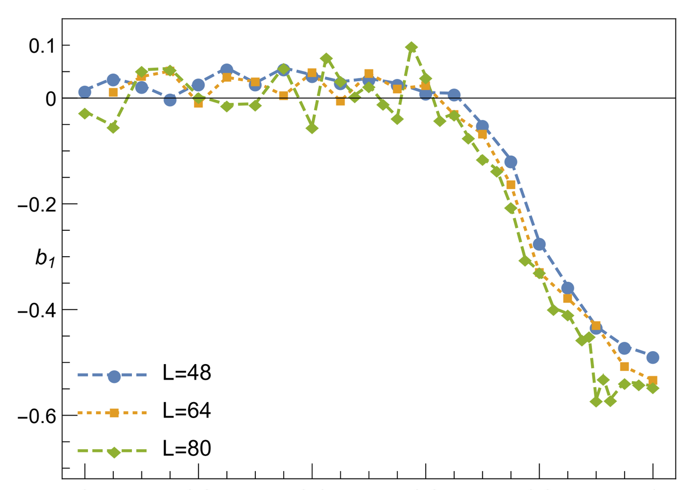

Figure 3:The skewness of distribution of critical load σ_c as a function of weight parameter g for different system sizes.

The distribution of critical load per pillar $\sigma_c = F_c/N$ can be fitted by skew normal distribution with probability density function:

$$p(\sigma_c) = \frac{\exp[-\frac{(\sigma_c-\xi)^2}{2\omega^2}]\mathrm{erfc}[-\frac{\alpha(\sigma_c-\xi)}{\sqrt{2}\omega}]}{\sqrt{2\pi}\omega} \qquad (2)$$

and cumulative distribution function

$$P(\sigma_c) = \frac{1}{2}\mathrm{erfc}(-\frac{\sigma_c - \xi}{\sqrt{2}\omega}) - 2\mathrm{T}(\frac{\sigma_c - \xi}{\omega}, \alpha) \qquad (3)$$

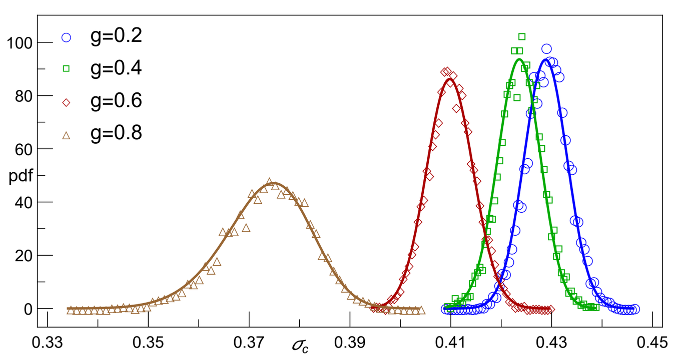

Figure 4:Empirical probability density functions of σ_c in the arrays of 80×80 pillars. The solid lines represent probability density functions of skew normally distributed σ_c with parameters ξ, ω, α computed from the samples.

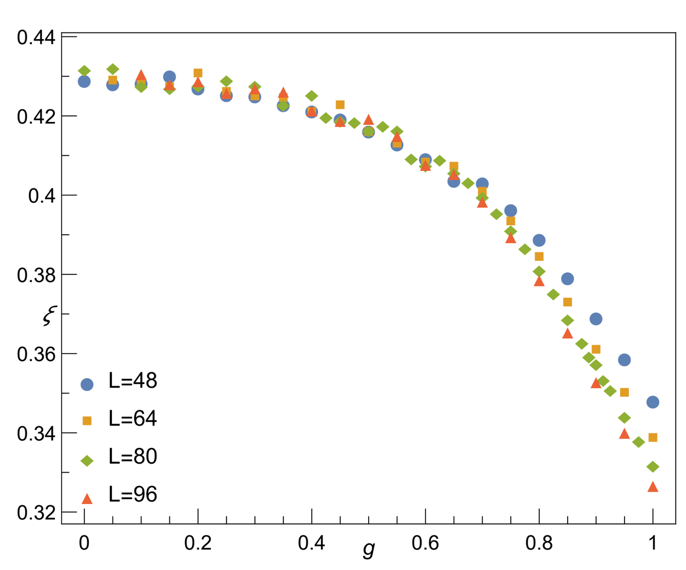

Figure 5:The location parameter ξ of skew normal distribution as a function of g for different system sizes.

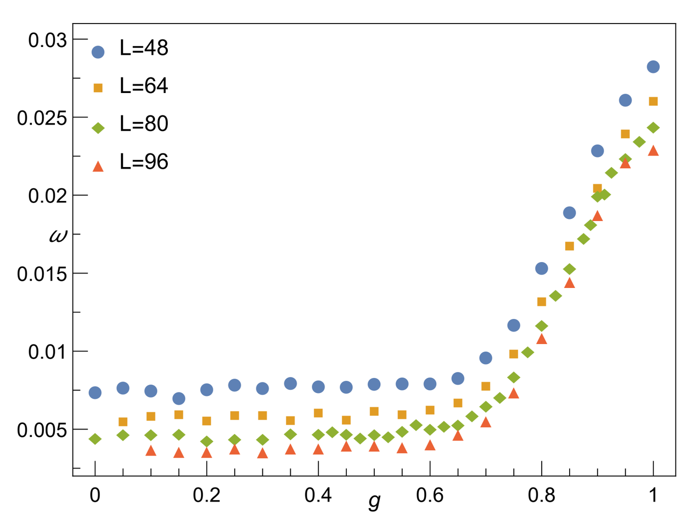

Figure 6:The scale parameter ω of skew normal distribution as a function of g for different system sizes.

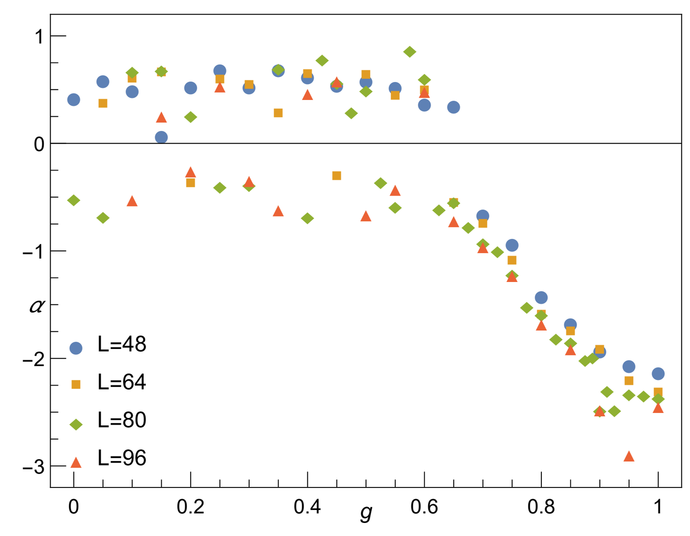

Figure 7:The shape parameter α of skew normal distribution as a function of g for different system sizes.

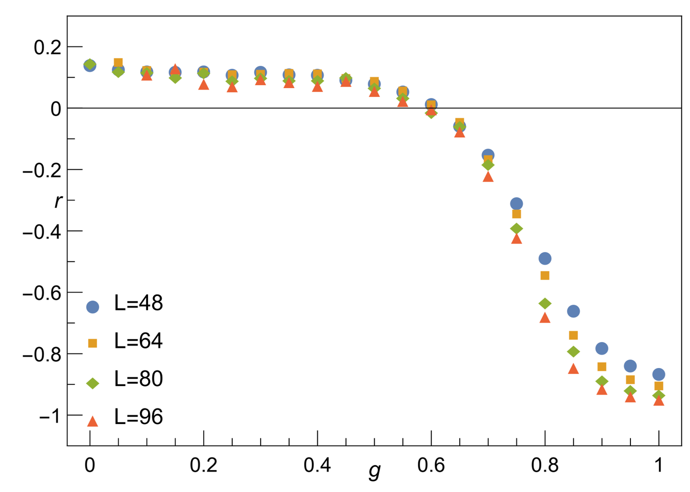

Figure 8:The correlation coefficient r between σ_c and critical avalanche Δ_c as a function of g for different system sizes.

Sudden loading

For fitting probability of breakdown we employ cumulative distribution function of skew normal distribution, and thus we rewrite formula (3)

$$P_b(\sigma) = \frac{1}{2}\mathrm{erfc}(-\frac{\sigma - \xi_{SL}}{\sqrt{2}\omega_{SL}}) - 2\mathrm{T}(\frac{\sigma - \xi_{SL}}{\omega_{SL}}, \alpha_{SL}) \qquad (4)$$

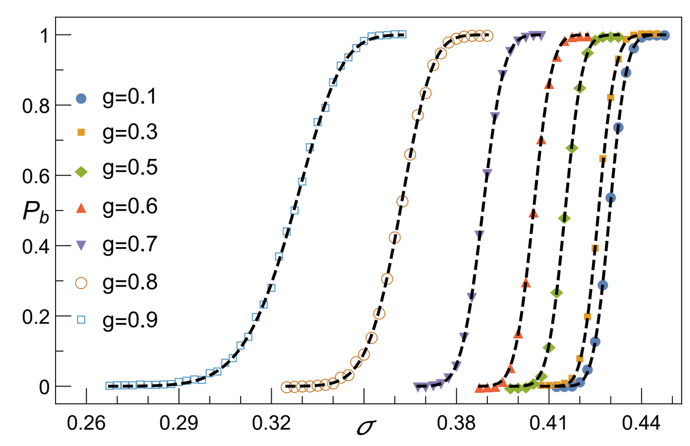

Figure 9:Empirical breakdown probability P_b as a function of initial load per pillar $\sigma = F/N$ for different values of weight parameter g. System size $N = 80 \times 80$. The dashed lines represent function (4) with parameters computed from simulations.

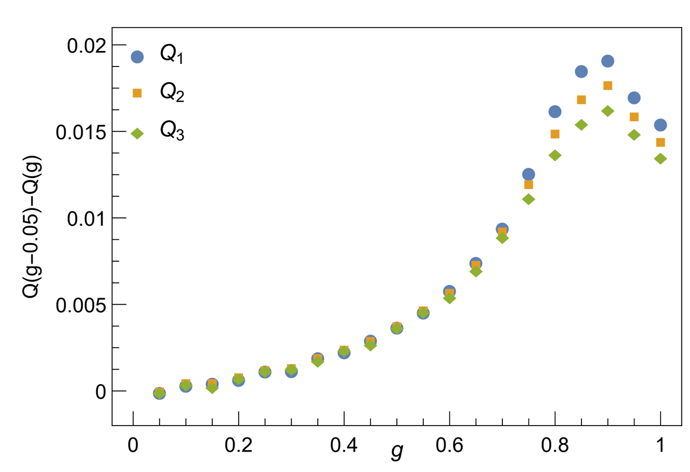

Figure 10:The results of subtracting quartile of SND for $g - 0.05$ from quartile of SND for g. Parameters of SND are based on the simulation results. The results concern systems with $N = 80 \times 80$ pillars.

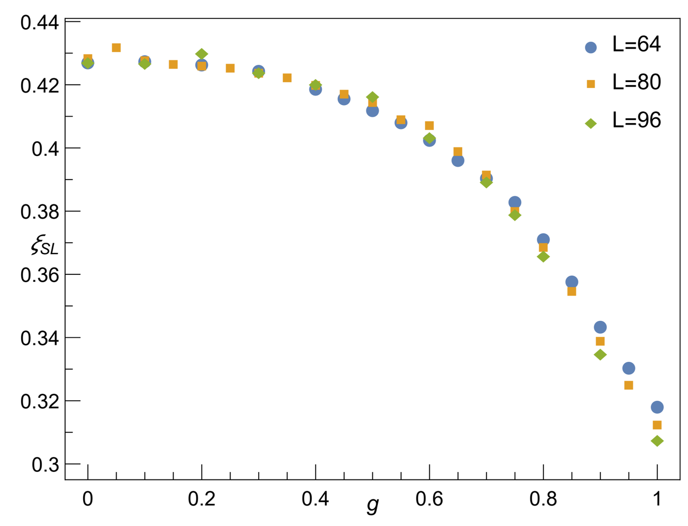

Figure 11:The parameter ξ_{SL} of formula (4) as a function of g for different system sizes.

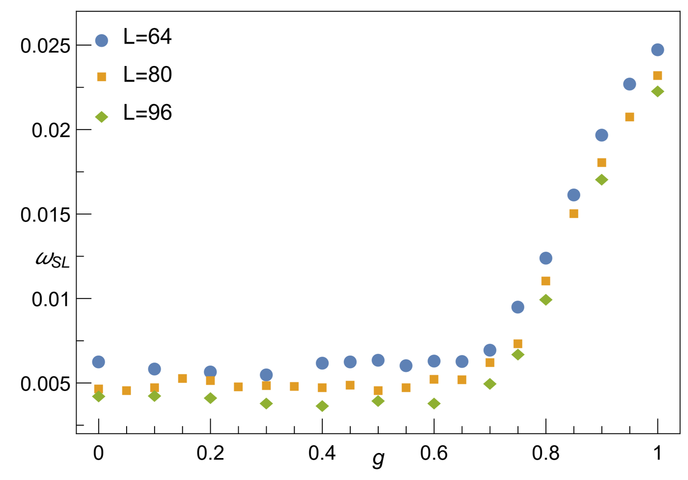

Figure 12:The parameter ω_{SL} of formula (4) as a function of g for different system sizes.

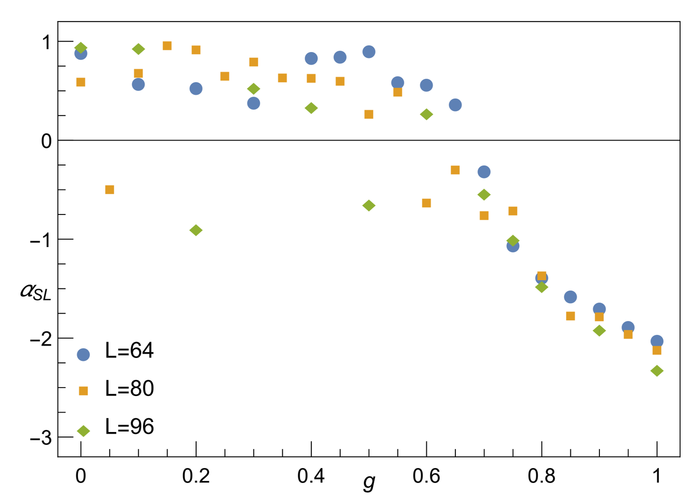

Figure 13:The parameter α_{SL} of formula (4) as a function of g for different system sizes.

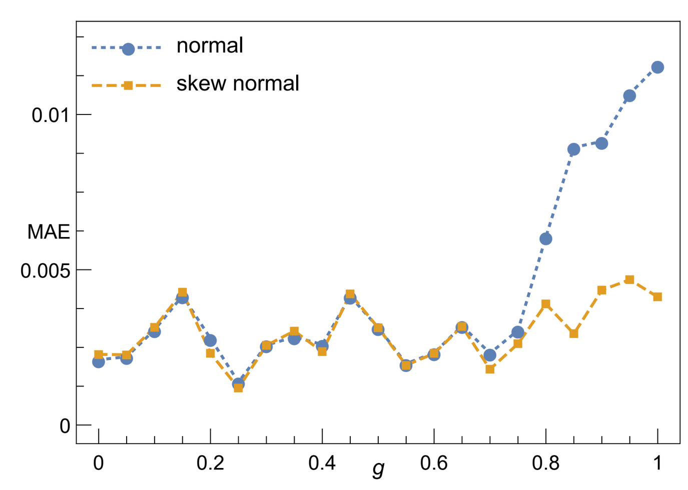

Figure 14:Mean absolute errors of P_b approximation using: CDF of the SND and CDF of the normal distribution. The results concern systems with $N = 80 \times 80$ pillars.

References

[1] A. Hansen, P.C. Hemmer, S. Pradhan, "The Fiber Bundle Model: Modeling Failure in Materials", Wiley, 2015.

[2] M.J. Alava, P.K.V.V. Nukala, S. Zapperi, "Statistical models of fracture", *Adv. In Physics*, vol. 55, pp.349-476, 2006.

[3] S. Pradhan, B.K. Chakrabarti, S. Zapperi, "Crossover behavior in a mixed-mode fiber bundle modele", *Phys. Rev. E*, 71, 036149, 2005.

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